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An alternative class of supersymmetries

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Abstract. The supersymmetry of Wess and Zumino is generalized to square roots of a class of ordinary Lie groups. The case of $O_{2,3}$ is studied in detail. We obtain supersymmetric equations of motion for fields on a variant of de-Sitter space. The only dimensional parameter is the radius of space–time.

1. Introduction

A transformation group mixing boson and fermion fields on space–time was recently invented by Wess and Zumino (1974). Its properties have been studied notably by Salam and Strathdee (1974a, b, c), and the quantum field theory using invariant Lagrangians by Iliopoulis and Zumino (1974).

This paper is concerned with another group that may be used to mix bosons and fermions. Wess and Zumino's group is in a sense the square root of the group T_4 of space–time translations. We give a simple definition for square roots of an ordinary Lie group. We find a method for determining whether any exist, and their structure. We study in particular a square root of the group $O_{2,3}$.

$\sqrt{T_4}$ has the particularly simple and useful property that its commutant lies in its centre: the commutators of the infinitesimal generators are all translation generators, and translations commute with all elements of $\sqrt{T_4}$. $\sqrt{O_{2,3}}$ does not share this property, and some of the techniques that work for $\sqrt{T_4}$ do not work for $\sqrt{O_{2,3}}$.

A departure from common practice is that space–time is taken to be a variant of de-Sitter space: a hyperboloid embedded in a five-dimensional space. Thus, we live near the point $(0, 0, 0, 0, R)$, say, on

$$x_0^2-x_1^2-x_2^2-x_3^2+x_4^2 = R^2$$

and R is large. Maxwell and Dirac equations for this space were obtained by Dirac (1935, see also Börner and Dürr 1969 for later references). Instead of the space–time symmetry group being the Poincaré group, it is $O_{2,3}$, the former being a contraction related to $R \to \infty$ of the latter. We should perhaps take the universal covering space of this hyperboloid, since otherwise an observer in free fall would find himself at the same space–time point after a proper time $2\pi R$ (see for example Hawking and Ellis 1973). Everything we say can be said for either space–time. For calculations it is more convenient to work embedded in five-space.

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In § 2 we establish the existence of $\sqrt{O_{2,3}}$, and find some simple finite-dimensional representations in § 3. Using Salam and Strathdee's (1974b) method of superfields we find two representations on fields in § 4, and exhibit a Lagrangian in the final section.

The distinctive feature is that not only does the supersymmetry mix fermion terms with boson terms in the Lagrangian, but kinetic terms with mass terms. For $R \to \infty$ we have a massless spinor (two-component) field and a massless vector field.

2. A class of supersymmetries

As usual (Wess and Zumino 1974), the generators of our group† G fall into two classes: (i) for each real multiplet α of anticommuting numbers (a numbers) a generator T_α (supergenerators); (ii) the generators T_ξ of a subgroup H, essentially a finite-dimensional Lie group, where the ξ are a set of commuting (c number) real parameters: $\xi = (\xi^A)$, $T_\xi = T_A\xi^A$.

H mixes the generators T_α among themselves:

$$[T_\xi, T_\alpha] = T_{\xi . M\alpha}$$

where $\xi . M = \xi^A M_A$, the M_A being matrices (with real c number elements) representing the T_A in α space.

The multiplication table is completed by specifying

$$[T_\alpha, T_\beta] = T_\xi$$

where ξ is linear and antisymmetric in α and β. Thus $\xi^A = \mathrm{i}\alpha^T\Delta^A\beta$ where the Δ^A are real symmetric matrices. They must also transform appropriately under H: Δ^A_{ij} must be an invariant H tensor. There remains one condition for the above to define a group, namely the Jacobi identity

$$[[T_\alpha, T_\beta], T_\gamma] + [[T_\beta, T_\gamma], T_\alpha] + [[T_\gamma, T_\alpha], T_\beta] = 0.$$

This is satisfied if and only if

$$M_{Ai}{}^{j}\Delta^{Akl} + M_{Ai}{}^{k}\Delta^{Alj} + M_{Ai}{}^{l}\Delta^{Ajk} = 0.$$

This may be written more usefully

$$M_{Ai}{}^{j}\Delta^{Akl} = R_i{}^{kjl} + R_i{}^{ljk} \tag{1}$$

where $R_i{}^{jkl}$ is antisymmetric under interchange of k and l. Clearly $R_i{}^{jkl}$ must be an invariant H tensor.

We now see how to construct square root groups G or H. The prescription is:

(i) take a real representation of H;
(ii) find an H invariant tensor $R^i{}_{jkl}$, antisymmetric on the last two indices;
(iii) solve equation (1).

Of course either of (ii) and (iii) may be impossible. (ii) and (iii) will probably ensure that Δ^{Aij} is an invariant tensor, though this may not be so if the solutions of (1) are not unique.

In practice we are interested in G whose relation to the Lorentz group is such that α has half-odd-integer spin. In this paper we consider the simplest case, namely that

† We do not distinguish groups from algebras in notation.

α is a Majorana spinor. α being real, we use the Majorana representation of the Dirac matrices, so γ^5, $i\gamma^\mu$, and $i\sigma^{\mu\nu}$ are real.

It follows that R_i^{jkl} must be Lorentz invariant and we consider the simplest case $\delta_i^j(iC^{-1})^{kl}$. (The tensor $\delta_i^k(iC^{-1})^{jl}-\delta_i^l(iC^{-1})^{jk}$ gives the same right-hand side to (1).) For Wess and Zumino, R_i^{jkl} actually vanishes. We suppose that it does not. Equation (1) is then a completeness relation for the space of real symmetric four by four matrices, so both $M_A iC$ and Δ^A span this space†. If we make the further assumption that H is represented faithfully (at least locally) on α space it follows that the M_A may be taken to be proportional to $i\gamma^\mu$ and $i\sigma^{\mu\nu}$. If we put $\sigma^{4\mu} = \gamma^\mu$, $g_{44} = 1$, and use Latin indices $a, b, \ldots$ for 0, 1, 2, 3, 4 we have

$$[\sigma_{ab}, \sigma_{cd}] = 2i(\sigma_{ad}g_{bc}+\sigma_{bc}g_{ad}-\sigma_{ac}g_{bd}-\sigma_{bd}g_{ac})$$

so H is $O_{2,3}$‡. The real matrices $\kappa^a = (i\gamma^\mu\gamma^5, \gamma^5)$ form a vector multiplet and with $(i\sigma^{ab})$ and the unit matrix form a complete set of real four by four matrices.

We have $T_\xi = \frac{1}{2}\xi^{ab}T_{ab}$, and if we normalize the T_{ab} such that $\delta V^a = \xi^a{}_b V^b$ for a vector V^a then $M_{ab} = -\frac{1}{2}i\sigma_{ab}$ and

$$[T_{ab}, T_{cd}] = T_{ad}g_{bc}+T_{bc}g_{ad}-T_{ac}g_{bd}-T_{bd}g_{ac}.$$

The other commutators are then

$$[T_\xi, T_\alpha] = T_\beta$$

where $\beta = -\frac{1}{4}i\sigma^{ab}\xi_{ab}\alpha$, and

$$[T_\alpha, T_\beta] = T_\xi \tag{2}$$

where $\xi_{ab} = \lambda\bar{\alpha}i\sigma_{ab}\beta$. We could take $\lambda = \pm 1$ without loss, and in fact the sign appears to be unimportant (for instance, λ is absent from the field equations of motion (7)).

Note that (2) means that contraction to Wess and Zumino's group, or any group with $T_\alpha \neq 0$, is not possible.

It is not difficult to show that G has only one Casimir operator (of any degree),

$$\mathbb{C} = T_iT_j(C^{-1})^{ij}-\lambda T_{ab}T^{ab}$$

where T_i is defined by $T_\alpha = (\bar{\alpha})^i T_i$.

3. Finite-dimensional representations

A simple action of G is that on G/H. We express the elements of G as $e^{T_\alpha}e^{T_\xi}$ and specify cosets of G/H by the elements e^{T_θ} (θ a multiplet of a numbers). θ transforms linearly under $O_{2,3}$, non-linearly under supertransformations. Because powers of θ greater than four vanish, the supertransformations can be calculated quite easily as follows. We have $\theta \to \theta'$ defined by

$$e^{T_\alpha}e^{T_\theta} = e^{T_{\theta'}}h$$

† This argument can be made for $2n \times 2n$ matrices for any n, and shows that one can take the square root of all the groups Sp_{nR}. A similar argument using complex representations of H leads to square roots of the pseudo-unitary groups $U_{m,n}$. Wess and Zumino actually gave the square root of the product of the conformal group and γ^5 transformations. This is the case $m = n = 2$.

‡ If we relax the reality condition on the a numbers, we can take instead γ^μ and $i\sigma^{\mu\nu}$, in which case H is $O_{1,4}$, the symmetry of de-Sitter space itself. The number of fields in the Salam–Strathdee superfield is then 256, instead of 16 for the present case. This is why we study $O_{2,3}$.

where $h \in H$. The automorphism $T_\alpha \to -T_\alpha$, $T_\xi \to T_\xi$ allows us to eliminate h:

$$e^{2T_{\theta'}} = e^{T_\alpha} e^{2T_\theta} e^{T_\alpha}$$

or

$$e^{-2T_\theta} \delta(e^{2T_\theta}) = T_\alpha + e^{-2T_\theta} T_\alpha e^{2T_\theta}.$$

Both left- and right-hand sides can be easily calculated in terms of θ, $\delta\theta$ and α using the commutation relations†. The result is

$$\delta\theta = [1 + \tfrac{1}{12}\lambda(5\bar{\theta}\theta - \bar{\theta}\kappa^a\theta \,.\, \kappa_a) - (\tfrac{1}{6}\lambda\bar{\theta}\theta)^2]\alpha. \tag{3}$$

By considering polynomials in θ we can obtain some simple finite-dimensional‡ (linear) representations of G. The polynomials

$$(1+\tfrac{1}{6}\lambda\bar{\theta}\theta)\theta \qquad \text{and} \qquad 1+\tfrac{1}{2}\lambda\bar{\theta}\theta+\tfrac{1}{24}\lambda^2(\bar{\theta}\theta)^2$$

form a multiplet, as do

$$\bar{\theta}\kappa^a\theta, (1+\tfrac{2}{3}\lambda\bar{\theta}\theta)\theta, \qquad \text{and} \qquad 1+\tfrac{3}{4}\lambda\bar{\theta}\theta+\tfrac{1}{4}\lambda^2(\bar{\theta}\theta)^2.$$

The corresponding representations are

$$\delta\psi = \lambda S\alpha, \qquad \delta S = \bar{\alpha}\psi$$

and

$$\delta V^a = 2\bar{\alpha}\kappa^a\psi, \qquad \delta\psi = (\tfrac{1}{4}\lambda V_a\kappa^a + S)\alpha, \qquad \delta S = \tfrac{3}{2}\lambda\bar{\alpha}\psi.$$

The former preserves the scalar product $\bar{\psi}\psi - \lambda S^2$, and the latter preserves

$$\bar{\psi}\psi - \tfrac{1}{8}\lambda V_a V^a - 2S^2/3\lambda.$$

This is in contrast to Wess and Zumino's group, for which the analogous action is $\delta\theta = \alpha$. There the space of polynomials is not even completely reducible, though the subspaces consisting of polynomials of given degree are invariant.

4. Fields

Another action of G is that on $G/O_{1,3}$, where $O_{1,3}$ is the subgroup of $O_{2,3}$ generated by the $T_{\mu\nu}$, ie the physical Lorentz group. The cosets may be specified by elements $e^{T_\theta} e^{T_y}$ where $T_y = y^\mu T_{4\mu}$.

Then θ transforms exactly as in § 3, independently of y.

† We have

$$e^{-A}(\delta e^A) = \sum_{n \geq 0} \frac{1}{(n+1)!}[\ldots[\delta A, A]\ldots A]$$

and

$$e^{-A} B e^A = \sum_{n \geq 0} \frac{1}{n!}[\ldots[B, A]\ldots A]$$

with n commutators in each case.

‡ Of course, really finite only if the a number space is finite dimensional.

Under $O_{2,3}$, y transforms as a point on the hyperboloid defined in the introduction†. Under supertransformations, $(\theta, y) \to (\theta', y')$ such that

$$e^{-T_{y'}} e^{-T_{\theta'}} e^{T_\alpha} e^{T_\theta} e^{T_y} = \text{Lorentz transformation}$$

or

$$(\delta e^{-T_y}) e^{T_y} + e^{-T_y}[e^{-T_\theta} T_\alpha e^{T_\theta} + (\delta e^{-T_\theta}) e^{T_\theta}] e^{T_y} = \text{Lorentz generator.}$$

The square bracket may be calculated using $\delta\theta$ from (3), and commutators. The result is T_ξ where

$$\xi_{ab} = \tfrac{1}{2}\lambda\bar{\alpha}\mathrm{i}\sigma_{ab}\theta \,.\, (1 - \tfrac{1}{12}\lambda\bar{\theta}\theta)$$

(its supercomponent must vanish). It remains to solve

$$(\delta e^{-T_y}) e^{T_y} + e^{-T_y} T_\xi e^{T_y} = \text{Lorentz generator.}$$

This is simply the transformation rule for y under e^{T_ξ}, so in terms of the coordinates x^a of five-space we have‡

$$\delta x^a = \tfrac{1}{2}\lambda\bar{\alpha}\mathrm{i}\sigma^{ab}\theta \,.\, (1 - \tfrac{1}{12}\lambda\bar{\theta}\theta) \,.\, x_b.$$

We can now use Salam and Strathdee's method for finding representations on fields. That is, we consider polynomials in θ whose coefficients are now functions of x:

$$\Phi(\theta, x) = S_1(x) + \bar{\theta}\psi_1(x) + \bar{\theta}\theta \,.\, S_2(x) + \bar{\theta}\kappa^a\theta \,.\, V_a(x) + \bar{\theta}\theta \,.\, \bar{\theta}\psi_2(x) + (\bar{\theta}\theta)^2 \,.\, S_3(x).$$

The field transformations are

$$\begin{aligned}
\delta S_1 &= -\bar{\alpha}\psi_1 \\
\delta S_2 &= -\bar{\alpha}[(\tfrac{5}{12}\lambda + \tfrac{1}{8}\lambda A)\psi_1 + \tfrac{1}{2}\psi_2] \\
\delta S_3 &= \bar{\alpha}[(\tfrac{1}{36}\lambda^2 + \tfrac{1}{96}\lambda^2 A)\psi_1 - (\tfrac{5}{12}\lambda + \tfrac{1}{8}\lambda A)\psi_2] \\
\delta\psi_1 &= -(2S_2 + 2x_a V^a + \tfrac{1}{2}\lambda A S_1)\alpha \\
\delta\psi_2 &= (\tfrac{1}{24}\lambda^2 A S_1 - \tfrac{1}{2}\lambda A S_2 - 4S_3 - \tfrac{4}{3}\lambda\kappa_a V^a + \lambda x_{[a}\partial_{b]}\kappa^a V^b \\
&\quad - \tfrac{1}{4}\lambda\epsilon^{abcde} x_a \partial_b \mathrm{i}\sigma_{cd} V_e)\alpha \\
\delta V^a &= \bar{\alpha}[(\tfrac{1}{12}\lambda\kappa^a + \tfrac{1}{4}\lambda x^{[ab]}\kappa_b - \tfrac{1}{16}\lambda\epsilon^{abcde} x_b \partial_c \mathrm{i}\sigma_{de})\psi_1 - \tfrac{1}{2}\kappa^a\psi_2]
\end{aligned}$$

† If instead of the hyperboloid we use its universal covering space, we replace $O_{2,3}$ by its universal covering.

‡ The connection between x and y may be calculated for example by using the spinor representation of T_{ab} and the automorphism $T_{4\mu} \to -T_{4\mu}$, $T_{\mu\nu} \to T_{\mu\nu}$.

The latter gives

$$\delta e^{2T_y} = \{T_\xi, e^{2T_y}\}.$$

The former gives

$$e^{2T_y} = \cos(|y^2|)^{1/2} - \mathrm{i}\gamma_\mu y^\mu \,.\, \frac{\sin(|y^2|)^{1/2}}{(|y^2|)^{1/2}}.$$

The result is

$$x^\mu = -R y^\mu \frac{\sin(|y^2|)^{1/2}}{(|y^2|)^{1/2}}.$$

where $x_{[a}\partial_{b]} = \frac{1}{2}(x_a\partial_b - x_b\partial_a)$, differential operators that act within the hyperboloid, $A \equiv i\sigma^{ab}x_a\partial_b$, and the alternating symbol is fixed by $\epsilon_{01234} = 1$.

This action may be reduced: if

$$S' = \tfrac{1}{8}\lambda S_1 - \tfrac{4}{3}S_2 + 4S_3/\lambda$$

$$\psi' = (\tfrac{2}{3} + \tfrac{1}{4}A)\psi_1 - \psi_2/\lambda$$

and

$$S_1'' = -\tfrac{1}{4}\lambda S_1 - \tfrac{2}{3}S_2 + 8S_3/\lambda$$

$$S_2'' = -\lambda S_1 + 2S_2$$

$$S_3'' = x_a V^a$$

$$S_4'' = \partial_a^\perp V^a$$

$$\psi'' = (\tfrac{1}{3} - \tfrac{1}{2}A)\psi_1 - 2\psi_2/\lambda$$

then

$$\delta S' = \tfrac{1}{2}\lambda\bar{\alpha}(2 + A)\psi'$$

$$\delta\psi' = (S' + \tfrac{1}{2}\epsilon_{abcde}x^a\partial^b V^c i\sigma^{de})$$

$$\delta V^a = \tfrac{1}{2}\lambda\bar{\alpha}\kappa^a\psi' + \lambda\bar{\alpha}\kappa_b[-\tfrac{1}{4}\partial_\perp^a x^b + \tfrac{1}{8}x^a(-x^b + \partial_\perp^b)]\psi_1$$

and

$$\delta S_1'' = \tfrac{1}{2}\lambda\bar{\alpha}(3 + A)\psi''$$

$$\delta S_2'' = \tfrac{1}{2}\lambda\bar{\alpha}\psi''$$

$$\delta S_3'' = \tfrac{1}{4}\lambda\bar{\alpha}\kappa_a x^a\psi''$$

$$\delta S_4'' = \tfrac{1}{4}\lambda\bar{\alpha}\kappa_a\partial_\perp^a\psi''$$

$$\delta\psi'' = (S_1'' + AS_2'' + 2\kappa_a(x^a + \partial_\perp^a)S_3'' - 2\kappa_a x^a S_4'')\alpha.$$

Here

$$\partial_a^\perp = \partial_a - \frac{1}{R^2}x_a \cdot x\partial \qquad \text{and} \qquad \Box = \partial_a^\perp\partial_\perp^a.$$

With an appropriate decomposition of V^a, each of these new multiplets (called vector and scalar respectively) is invariant. We use

$$V^a = V_{\parallel}^a + x^a S_3'' + \partial_\perp^a\frac{1}{\Box}(S_4'' - 4S_3'')$$

so $\partial_a V_{\parallel}^a = x_a V_{\parallel}^a = 0$, and

$$T_{ab} = \left(\partial_a^\perp - \frac{1}{R^2}x^a\right)V_b^\perp - \left(\partial_b^\perp - \frac{1}{R^2}x_b\right)V_a^\perp$$

so $x^a T_{ab} = 0$ and

$$\left(\partial_a^\perp - \frac{2}{R^2}x_a\right)T_{bc} + \left(\partial_b^\perp - \frac{2}{R^2}x_b\right)T_{ca} + \left(\partial_c^\perp - \frac{2}{R^2}x_c\right)T_{ab} = 0. \tag{4}$$

In intrinsic coordinates this corresponds to the familiar

$$T_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \qquad \text{and} \qquad T_{\mu\nu,\lambda} + T_{\lambda\mu,\nu} + T_{\nu\lambda,\mu} = 0.$$

$V^a_{\parallel}$ and T_{ab} contain the same information.

This reduction has the property that the action remains local in the sense that the small changes in the new fields involve a finite number of derivatives of new fields, except for $V^a_{\parallel}$.

So far as Lorentz behaviour is concerned, the two multiplets are the same as occur in the reduction of the Hermitian superfield for Wess and Zumino's group.

The Casimir operator is simply $2\lambda[(2/R^2) - \Box]$ on the vector multiplet. On the scalar multiplet $\mathbb{C}$ is not a function of $\Box$. The scalar multiplet may itself be reduced into two multiplets, on each of which $\mathbb{C}$ is a function of $\Box$, but on which the action is non-local.

The reader may ask why it is that some such elegant procedure (Salam and Strathdee 1974a) as reduces the Wess–Zumino superfield cannot be used here. The essence of this procedure is the existence of a family of transformations of (θ, x) that commute with the group action. The group action is (for Wess and Zumino)

$$m : g \to mg; \qquad m, g \in G$$

that is, left multiplication. Left multiplication by m commutes with right multiplication by n, for all $m, n \in G$. The covariant derivative is the generator of these transformations on superfields. In the present case the group action is

$$m : gH \to mgH.$$

The only transformation commuting with this is

$$(\theta, x) \to (\theta, -x)$$

corresponding to

$$gH \to gHa$$

where a is diagonal $(-1, 1, 1, 1, -1)$, say. This is not helpful in reducing the superfield.

5. Dynamics

A little algebra shows, using the constraints (4) on T_{ab}, that if

$$\mathscr{L} = \frac{1}{R}\bar{\psi}'(\tfrac{1}{2}A + 1)\psi' - \frac{1}{\lambda R}S'^2 - \frac{R}{2\lambda}T_{ab}T^{ab} \tag{5}$$

then

$$\delta\mathscr{L} = x_{[a}\partial_{b]}(\overline{\delta\psi'}\mathrm{i}\sigma^{ab}\psi' + \lambda S'\bar{\alpha}\mathrm{i}\sigma^{ab}\psi') + \partial_a^\perp(2\lambda\bar{\alpha}\kappa_b\psi' \,.\, T^{ab}).$$

Stokes' theorem says that if $x^a f_a(x) = 0$ then $\int \mathrm{d}\sigma\, \partial_a^\perp f^a = 0$, where $\mathrm{d}\sigma$ is the $O_{2,3}$ invariant measure on the hyperboloid. Also, if $f^{ab}(x)$ is antisymmetric, then

$$\partial_a^\perp(f^{ab}x_b) = x_{[a}\partial_{b]}f^{ab}.$$

So the equations of motion derived from $\mathscr{L}$ are supersymmetric.

$\mathscr{L}$ is the only quadratic invariant up to a derivative that one can construct with the vector multiplet if one uses only first derivatives of ψ'. There are none for the scalar multiplet if only first derivatives are used. There is just one for the original (reducible) multiplet if only first derivatives of ψ_1 and ψ_2 are used, namely

$$9\lambda S_1^2 - 40\, S_1 S_2 + \frac{48}{\lambda} S_1 S_3 + \frac{24}{\lambda} S_2^2 + 5\bar{\psi}_1\psi_1 - \frac{12}{\lambda}\bar{\psi}_1\psi_2 + \frac{24}{\lambda} V_a V^a. \tag{6}$$

No derivatives occur. This expression may be obtained by a trick as follows. It is easy to see that the variation of

$$9\lambda S_1 - 20\, S_2 + \frac{24}{\lambda} S_3$$

is a derivative. Our expression (6) is the corresponding expression for Φ^2.

Dirac (1935) suggested the Lagrangian

$$\tfrac{1}{2}\bar{\psi}\left(\frac{1}{R}A - m\right)\psi$$

with real m. Near $(0, 0, 0, 0, R)$, and for ψ oscillating over distances of the order $1/m$ or less, this is approximately

$$\tfrac{1}{2}\bar{\psi}(\mathrm{i}\,\partial\!\!\!/ - m)\psi$$

so m is the mass.

$\mathscr{L}$ gives, for $R \to \infty$, a massless spinor field and a massless vector field. It is not sensible to ask what the masses are for finite R, because the Casimir operator p^2 does not exist. One can see this as follows. The equations of motion

$$\begin{aligned} (\tfrac{1}{2}A + 1)\psi &= 0 \\ \partial_b^\perp T^{ab} &= 0 \end{aligned} \tag{7}$$

imply

$$\left(\Box - \frac{2}{R^2}\right)\psi = 0$$

$$\left(\Box - \frac{2}{R^2}\right)V^a = 0.$$

If we use the first four components of x^a as intrinsic coordinates we have

$$\Box = \left(\eta^{\mu\nu} - \frac{x^\mu x^\nu}{R^2}\right)\partial_{\mu\nu} - \frac{4}{R^2}x^\mu\partial_\mu.$$

Finally, we mention the question of interactions. There are no superficially renormalizable and supersymmetric interactions for vector multiplets. However, the work of Ferrara and Zumino (1974) and Salam and Strathdee (1974c) on combining internal symmetries with supersymmetries (Wess and Zumino's, and supergauge) shows how the situation may be more complicated. There the interaction is only superficially renormalizable in a particular gauge, which breaks the Wess–Zumino supersymmetry as well as the supergauge symmetry. A curious point is that, in this gauge, the quadratic part of their Lagrangian is exactly the same as the $R \to \infty$ limit of our Lagrangian (5) (with appropriate rescaling of S' and T_{ab}).

To summarize, we have shown the existence of an alternative class of supersymmetries, one of which has a possible physical application, and contains a mechanism for providing even stronger relations between coupling constants than occur with the supersymmetry of Wess and Zumino. As is the case with the latter, the outstanding problem is to find an application in which the fields can be physically identified.

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